How and to what extent does the spatial and temporal discretization schema affect GIS-based hydrological modelling?

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ABSTRACT
The justification of the spatial and temporal discretization schema is a critical step in the development of numerical hydrological models. Currently, the challenge remains in balancing the error and uncertainty induced by the algorithm and the mass calculation caused by the increase of the division of computational units. Thus, it is necessary to investigate an appropriate discretization scheme, which not only adequately represents the spatial heterogeneity characteristics, but also maintains a sufficiently high computational efficiency, with the constraints of the data validity and availability.

This poster paper proposed a numerical hydrological model using different spatial and temporal discretization schema. Results show that the running time revealed an increase by an order of magnitude with the refinement of the grid size. The results also show that that the discretization schema impose various influences on different hydrological processes. For the infiltration process, the effect of the spatial and temporal resolution depend on the soil type; for the runoff process, the amount of the runoff was less affected but the time to runoff was significantly influenced. Establishing a standardized method to optimize the range of the spatial-temporal resolution for different the models and environmental scenarios, however, still remains challenge and is the future investigations.

CCS CONCEPTS
•Information systems → spatial temporal systems

KEYWORDS
Spatial and temporal resolution, discretization schema, numerical simulation, discretization error

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1 Introduction
GIS-based hydrological models are widely used tools to simulate the hydrological response processes such as infiltration and runoff generation [1-3]. In such models, the determination of the discretization schema is critical. Generally speaking, the finer the resolution, the more accurate the simulation results will be [4]. However, Qiu et al. [5] and Politi et al. [6] reported that the accuracy of the simulation results with the three spatial resolution were not much different. Thus, it remains inconclusive whether finer resolution would achieve better modelling results in the numerical experiments with various conditions. Besides, the spatial and temporal discretization schema also have effects on the computational efficiency as different resolution may change the structure of the model and the number of iterations [7].

In addition, the choice of spatial and temporal resolution have different effects on the simulation depending on the type and nature of the hydrological response processes [8-9]. For the infiltration process, for example, it was found that the parameters related to soil water movement and the estimated infiltration amount at different resolution differed by several orders of magnitude [10]. For the runoff generation process, Unami et al. [11] reported that the simulated peak amount of the runoff was smoothed as the catchment is divided into fewer and larger sub-basins. It is also found that the spatiotemporal resolution is more sensitive to the soil and vegetation conditions, which exhibit more local variations, compared with the climatic conditions [12-13].

Therefore, our objectives are (1) to conduct a numerical hydrological model and investigate an appropriate discretization scheme with high computational efficiency and modelling stability; (2) to investigate the impact of different spatial and temporal discretization schema on the simulation results of various hydrological response process based on the numerical experiments. In this study, we adjust the temporal resolution and the grid size in the iterative process to obtain the optimal range under different
initial conditions, and quantify the influence of different spatial and temporal discretization schema on the simulation results.

2 Description of the numerical model

The hydrological process of infiltration and runoff were coupled in two main modules. The model has been conducted with the MATLAB programming language. The computer configuration included: the system type is Windows 10 64-bit operating system, the processor is Intel core i3 with 3.1GHz, and the installed RAM is 8G.

Lighthill and Whitham [14] proposed the kinematic wave equation as an approximate method of Saint-Venant’s equation to estimate one-dimensional surface flow. For shallow surface flow, ignoring the velocity and pressure head gradient in the momentum equation [15-16], the first-order hyperbolic partial differential equation can be obtained:

$$\frac{\partial A}{\partial t} + \frac{\partial Q}{\partial x} = i_e \cdot b$$  \hspace{1cm} (1)

Where $A$ is the area of the cross section; $b$ is the width of the cross section; $Q$ is the flow rate; $i_e$ is the excess rainfall (impermeable part); $t$ is the time; $x$ is the downslope distance.

Substituting $A = b \cdot h$ and $Q = q \cdot b$ into the equation (1), we can get:

$$\frac{\partial b \cdot h}{\partial t} + \frac{\partial q \cdot b}{\partial x} = i_e \cdot b$$  \hspace{1cm} (2)

For turbulent flow conditions, the slope roughness cannot be ignored, the Manning formula is used to express the relationship between and. Here it is assumed that the hydraulic gradient of the slope flow is equal to the angle of the slope, that is, the loss of runoff caused by surface filling is also considered:

$$q = \frac{1}{n} \sqrt{S_0 (h-d_s) \frac{5}{3}}$$  \hspace{1cm} (3)

Where $S_0$ is the slope angle, $n$ is the roughness, and $d_s$ is the amount of landfill savings.

The infiltration equation coupled with the hydrodynamic model was obtained by Mein and Larson [14]. Taking into account the influence of surface water, this equation was chosen considering the robustness and respect the data availability:

$$F(t) - F(t-\Delta t) = \Delta F = \xi \ln\left(\frac{F(t) + \xi}{F(t-\Delta t) + \xi}\right) + k_{sat} \Delta t$$  \hspace{1cm} (4)

$$\frac{\partial F}{\partial t} + \frac{\partial (Fq)}{\partial x} = k_{sat} \cdot A_i$$  \hspace{1cm} (5)

Where $F$ is the cumulative infiltration capacity, $K_{sat}$ is the saturated hydraulic conductivity of the soil, $\xi = \psi_s (\theta_s - \theta_i)$, $\psi_s$ is the soil water suction at the wet front, and $\theta_i$, $\theta_s$ is the saturated water content and the initial water content of the soil, respectively.

The second-order Newton method was used to determine the cumulative infiltration volume within the time increment $\Delta t$:

$$\frac{\Delta F}{\Delta t} = -F - \left(\frac{F + \Delta F_0}{F} \right) + \left(\frac{F + \xi + \Delta F_0}{F} \right) \left(\frac{F + \Delta F_0}{F + \xi} \right)^{\frac{1}{2}}$$

The numerical solution of the first-order partial differential equation is:

$$\frac{h_{k+1}^j - h_k^j}{\Delta t} + q_j^f \cdot \frac{f_{w_k} + f_{w_{k+1}}}{2}$$

$$\frac{q_j^f \cdot \frac{f_{w_k} + f_{w_{k+1}}}{2}}{\Delta x \cdot f_{w_k}^e} = \frac{\left( i_{ek}^j + i_{ek}^{j+1} \right)}{2}$$  \hspace{1cm} (6)

To get the numerical solution, a fully implicit schema was used to solve the Green-Ampt equation and the runoff module was solved with an explicit schema.

3 The input and designing of the numerical experiment

A homogenous and rectangular catchment was selected as the study area. This small catchment is 100 m long and 10 m. Three different grid sizes of 20cm, 10cm, and 5cm are selected; three different temporal resolution of 1.44min, 14.4min and 144min were performed in the numerical model. The input data and the value of the parameters in the model are presented in table 1.

<table>
<thead>
<tr>
<th>Table 1: Values of model parameters and input data</th>
</tr>
</thead>
<tbody>
<tr>
<td>Input and parameter</td>
</tr>
<tr>
<td>Temporal resolution</td>
</tr>
<tr>
<td>Grid size</td>
</tr>
<tr>
<td>Rainfall intensity</td>
</tr>
<tr>
<td>Soil bulk density</td>
</tr>
<tr>
<td>Saturated hydraulic conductivity</td>
</tr>
<tr>
<td>Initial suction</td>
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</tbody>
</table>
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<table>
<thead>
<tr>
<th>Manning coefficient</th>
<th>s/m</th>
<th>0.25</th>
</tr>
</thead>
<tbody>
<tr>
<td>slope</td>
<td>°</td>
<td>20,30,40</td>
</tr>
<tr>
<td>Depression storage and intercepting</td>
<td>mm</td>
<td>1</td>
</tr>
</tbody>
</table>

As for the soil hydraulic properties, three different soil hydraulic characteristic curves with different bulk density were used to drive the model, as shown in Figure 1, which is proposed with reference to the experimental data of Genuchten et al. [17].

![Figure 1: Soil hydraulic characteristic curve of with different bulk density (LC is the low bulk density soil, MC is the medium bulk density soil, and UP is the high bulk density soil).](image)

### 4 Results and discussion

As shown in table 2, the running time with different spatial resolution are presented, indicating that with the refinement of the grid size (from 20cm to 5cm), the running time shows an increase by an order of magnitude, which suggest that the determination of the grid size is of great significance to the computational efficiency.

<table>
<thead>
<tr>
<th>Grid size (cm)</th>
<th>Running time (s)</th>
<th>Accumulative runoff (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>74.28</td>
<td>12.28</td>
</tr>
<tr>
<td>10</td>
<td>512.23</td>
<td>12.435</td>
</tr>
<tr>
<td>5</td>
<td>3251.75</td>
<td>13.47</td>
</tr>
</tbody>
</table>

For the estimated accumulative runoff in table 2, the numerical solutions under three different spatial resolution are stable and there is no discretization oscillation. When the grid size become finer, the simulated runoff increased gently. The discharge hydrograph with three grid size are as shown in the Figure 2. At three different grid resolution, the peak flow and the streamflow shape withdrawal consistent trend curve, but with different grid size the runoff amount is different.

![Figure 2: the discharge hydrograph of the runoff simulation results under three different spatial resolution.](image)

As shown in Figure 3, the simulation of time to runoff can be affected by many factors, including the environmental factors, such as rainfall intensity, soil properties, slope and soil moisture content, and factors related to the model itself, such as the discretization schema, the equations used in the model and so on. When the temporal resolution takes different values from 1.44min to 144min, the simulation results varies with different soil bulk density. For low bulk density soil, the change of the temporal resolution does not have much effect on the simulation of time to runoff, and the estimated response time is almost the same. However, for the low and medium bulk density soils, when the temporal resolution increased to 144min, abnormal values were found in the simulation results, which suggests that the numerical solutions with the temporal resolution of 144min does not converge.

![Figure 3: Simulation of pressure heads with different soil properties under three different temporal resolution with the same grid size of 10cm.](image)

Figure 4 illustrates that the infiltration rate computed for different spatial resolution. For the high bulk density soil, the infiltration...
runoff was less affected but the time to runoff was significantly influenced by the spatial and temporal resolution. However, the challenge remains, to establish a standardized method to optimize the range of the spatial-temporal resolution for different the models and environmental scenarios, which is also the subject to our future investigations.

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REFERENCES
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5 Conclusion
This study developed an infiltration-runoff numerical model coupling the Green-Ampt equation and the kinematic wave equation. Numerical experiments were implemented using different spatial and temporal discretization schema. It was found that the running time showed an increase by an order of magnitude with the refinement of the grid size from 20 to 5 cm. Results showed the discretization schema impose various influences on different processes. For the infiltration process, the effect of the spatial and temporal resolution depend on the soil type. Under the low bulk density soil with large hydraulic conductivity, the simulation results were much more sensitive than under the low hydraulic conductivity. For the runoff process, the amount of the runoff was less affected but the time to runoff was significantly influenced by the spatial and temporal resolution. However, the challenge remains, to establish a standardized method to optimize the range of the spatial-temporal resolution for different the models and environmental scenarios, which is also the subject to our future investigations.

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